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ON A HEURISTIC POINT OF VIEW CONCERNING THE PRODUCTION
AND TRANSFORMATION OF LIGHT

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[1]

There exists a profound formal difference between the theoretical conceptions physicists have formed about gases and other ponderable bodies, and Maxwell's theory of electromagnetic processes in so-called empty space. While we conceive of the state of a body as being completely determined by the positions and velocities of a very large but nevertheless finite number of atoms and electrons, we use continuous spatial functions to determine the electromagnetic state of a space, so that a finite number of quantities cannot be considered as sufficient for the complete description of the electromagnetic state of a space. According to Maxwell's theory, energy is to be considered as a continuous spatial function for all purely electromagnetic phenomena, hence also for light, while according to the current conceptions of physicists the energy of a ponderable body is to be described as a sum extending over the atoms and electrons. The energy of a ponderable body cannot be broken up into arbitrarily many, arbitrarily small parts, while according to Maxwell's theory (or, more generally, according to any wave theory) the energy of a light ray emitted from a point source of light spreads continuously over a steadily increasing volume.

[2]

[3]

The wave theory of light, which operates with continuous spatial functions, has proved itself splendidly in describing purely optical phenomena and will probably never be replaced by another theory. One should keep in mind, however, that optical observations apply to time averages and not to momentary values, and it is conceivable that despite the complete confirmation of the theories of diffraction, reflection, refraction, dispersion, etc., by experiment, the theory of light, which operates with continuous spatial functions, may lead to contradictions with experience when it is applied to the phenomena of production and transformation of light.

[4]

Indeed, it seems to me that the observations regarding "black-body radiation," photoluminescence, production of cathode rays by ultraviolet

light, and other groups of phenomena associated with the production or conversion of light can be understood better if one assumes that the energy of light is discontinuously distributed in space. According to the assumption to be contemplated here, when a light ray is spreading from a point, the energy is not distributed continuously over ever-increasing spaces, but consists of a finite number of energy quanta that are localized in points in space, move without dividing, and can be absorbed or generated only as a whole. [5]

In this paper I wish to communicate my train of thought and present the facts that led me to this course, in the hope that the point of view to be elaborated may prove of use to some researchers in their investigations.

§1. On a difficulty encountered in the theory of "black-body radiation"

We shall begin by taking the standpoint of Maxwell's theory and the electron theory and consider the following case. Consider a space enclosed by completely reflecting walls containing a number of gas molecules and electrons that move freely and exert conservative forces on each other when they come very close to each other, i.e., they can collide like gas molecules according to the kinetic theory of gases.¹ Suppose, further, that a number of electrons are bound to points in space which are very far from each other, by forces that are directed toward these points and are proportional to the elongations from the points. These electrons, too, shall enter into conservative interactions with the free molecules and electrons when the latter come very close to them. We call the electrons bound to the points in space "resonators"; they emit and absorb electromagnetic waves of definite periods. [7]

According to the present view about the origin of light, the radiation in the space considered, found for the case of dynamic equilibrium on the basis of Maxwell's theory, must be identical with "black-body radiation" — at least if one assumes that resonators of all the relevant frequencies are present. [8] [9]

¹This assumption is equivalent to the assumption that the mean kinetic energies of gas molecules and electrons are equal to each other at thermal equilibrium. As we know, Mr. Drude used the latter assumption to derive the ratio of thermal and electric conductivities of the metals theoretically. [6]

For the time being, we disregard the radiation emitted and absorbed by the resonators and look for the condition for dynamic equilibrium corresponding to the interaction (collisions) of molecules and electrons. For such an equilibrium, the kinetic theory of gases provides the condition that the mean kinetic energy of a resonator electron must be equal to the mean kinetic energy of the progressive motion of a gas molecule. If we resolve the motion of the resonator electron into three mutually perpendicular oscillatory motions, we find for the mean value \bar{E} of the energy of such a linear oscillatory motion

$$\bar{E} = \frac{R}{N} T ,$$

[10] where R denotes the universal gas constant, N the number of "real molecules" in one gram-equivalent, and T the absolute temperature, for because of the equality of the time averages of the resonator's kinetic and potential energies, the energy \bar{E} is 2/3 times as large as the kinetic energy of a free monoatomic gas molecule. If due to some factor--in our case, due to radiation--the energy of a resonator were to have a time average larger or smaller than \bar{E} , the collisions with the free electrons and molecules would lead to an energy transfer to the gas or an energy absorption from the gas that is, on average, different from zero. Thus, in the case we are considering, dynamic equilibrium is possible only if the mean energy of every resonator equals \bar{E} .

We now apply similar reasoning to the interaction between the resonators and the radiation present in the space. Mr. Planck has derived the condition for the dynamic equilibrium in this case¹ using the assumption that

[11] ¹M. Planck, *Ann. d. Phys.* 1 (1900): 99.

the radiation may be considered as the most disordered process imaginable.² He found

$$\bar{E}_\nu = \frac{L^3}{8\pi\nu^2} \rho_\nu . \quad [14]$$

\bar{E}_ν is here the mean energy of a resonator with the proper frequency ν (per oscillation component), L the velocity of light, ν the frequency, and $\rho_\nu d\nu$ the energy per unit volume of that part of the radiation whose frequency lies between ν and $\nu + d\nu$.

If, on the whole, the radiation energy of frequency ν does not continually decrease or increase, we must have

$$\frac{R}{N} T = \bar{E} = \bar{E}_\nu = \frac{L^3}{8\pi\nu^2} \rho_\nu ,$$

$$\rho_\nu = \frac{R}{N} \frac{8\pi\nu^2}{L^3} T . \quad [15]$$

²This assumption can be formulated as follows. We expand the Z -component of the electrical force (Z) at an arbitrary point of the space considered in a time interval between $t = 0$ and $t = T$ (where T shall denote a time period that is very large relative to all pertinent oscillation periods) in a Fourier series [12]

$$Z = \sum_{\nu=1}^{\nu=\infty} A_\nu \sin(2\pi\nu \frac{t}{T} + \alpha_\nu) ,$$

where $A_\nu \geq 0$ and $0 \leq \alpha_\nu \leq 2\pi$. If one imagines that at the same point in space such an expansion is made arbitrarily often at randomly chosen initial points of time, then one will obtain different sets of values for the quantities A_ν and α_ν . For the frequency of occurrence of the various combinations of values of the quantities A_ν and α_ν , there will exist, then, (statistical) probabilities dW of the form

$$dW = f(A_1 A_2 \dots \alpha_1 \alpha_2 \dots) dA_1 dA_2 \dots d\alpha_1 d\alpha_2 \dots$$

The radiation is in the most disordered state imaginable when

$$f(A_1, A_2 \dots \alpha_1, \alpha_2 \dots) = F_1(A_1) F_2(A_2) \dots f_1(\alpha_1) \cdot f_2(\alpha_2) \dots ,$$

i.e., when the probability of a specific value of one of the quantities A or α is independent of the values taken by the other quantities A and α , respectively. Hence, the more closely fulfilled the condition that the individual pairs of quantities A_ν and α_ν depend on the emission and absorption processes of *particular* groups of resonators, the closer to a "most disordered state imaginable" the radiation is to be viewed in our case. [13]

[16] This relation, obtained as the condition of dynamic equilibrium, not only fails to agree with experience but it also states that in our model a definite distribution of energy between ether and matter is out of the question, since the wider the chosen range of the resonators' frequencies, the larger the radiation energy of the space, and we obtain in the limit

$$[17] \quad \int_0^\infty \rho_\nu d\nu = \frac{R}{N} \frac{8\pi}{L^3} T \int_0^\infty \nu^2 d\nu = \infty .$$

[18] §2. *On Planck's determination of the elementary quanta*

We now wish to show that Mr. Planck's determination of the elementary quanta is to some extent independent of his theory of "black-body radiation."

Planck's formula¹ for ρ_ν , which has been sufficient to account for all observations made so far, reads

$$[20] \quad \rho_\nu = \frac{\alpha \nu^3}{\beta_\nu} \frac{1}{e^{\frac{\beta_\nu}{T}} - 1} ,$$

where

$$\alpha = 6.10 \times 10^{-56}$$

$$\beta = 4.866 \times 10^{-11} .$$

For large values of T/ν , i.e., for large wavelengths and radiation densities, this formula reduces in the limit to

$$\rho_\nu = \frac{\alpha}{\beta} \nu^2 T .$$

One can see that this formula agrees with that derived from the Maxwellian theory and the electron theory in §1. By equating the coefficients of the two formulas, we obtain

[19] ¹M. Planck, *Ann. d. Phys.* 4 (1901): 561.

$$\frac{R}{N} \frac{8\pi}{L^3} = \frac{\alpha}{\beta}$$

or

$$N = \frac{\beta}{\alpha} \frac{8\pi R}{L^3} = 6.17 \times 10^{23} ,$$

i.e., one atom of hydrogen weighs $1/N$ gram $= 1.62 \times 10^{-24}$ g. This is exactly the value found by Mr. Planck, which shows satisfactory agreement with values found for this quantity by other methods. [21]

We therefore arrive at the following conclusion: the greater the energy density and the wavelength of radiation, the more useful the theoretical principles we have been using prove to be; however, these principles fail completely in the case of small wavelengths and small radiation densities.

In the following, we shall consider "black-body radiation" in connection with experience without basing it on any model for the production and propagation of radiation.

§3. On the entropy of radiation

The following consideration is contained in a famous study by Mr. Wien and shall be presented here only for the sake of completeness. [22]

Consider radiation that occupies a volume v . We assume that the observable properties of this radiation are completely determined when the radiation density $\rho(\nu)$ is given for all frequencies.¹ Since radiations of different frequencies are to be viewed as separable from each other without expenditure of work and without supply of heat, the entropy of radiation can be represented in the form

$$S = v \int_0^{\infty} \varphi(\rho, \nu) d\nu ,$$

where φ is a function of the variables ρ and ν . One can reduce φ to a function of a single variable by formulating the assertion that adiabatic

¹This assumption is arbitrary. Naturally, we will maintain this simplest assumption as long as the experimental results do not force us to abandon it.

compression of radiation between reflecting walls does not change its entropy. However, we shall not enter into this, but will immediately investigate how the function φ can be obtained from the black-body radiation law.

In the case of "black-body radiation," ρ is such a function of ν that the entropy is a maximum at a given energy, i.e.,

$$\delta \int_0^\infty \varphi(\rho, \nu) d\nu = 0 ,$$

if

$$\delta \int_0^\infty \rho d\nu = 0 .$$

From this it follows that for every choice of $\delta\rho$ as function of ν

$$\int_0^\infty \left[\frac{\partial\varphi}{\partial\rho} - \lambda \right] \delta\rho d\nu = 0 ,$$

where λ is independent of ν . Thus for black-body radiation $\partial\varphi/\partial\rho$ is independent of ν .

The following equation applies when the temperature of black-body radiation of volume $v = 1$ increases by dT :

$$dS = \int_{\nu=0}^{\nu=\infty} \frac{\partial\varphi}{\partial\rho} d\rho d\nu ,$$

or, since $\partial\varphi/\partial\rho$ is independent of ν ,

$$dS = \frac{\partial\varphi}{\partial\rho} dE .$$

Since dE equals the heat added, and the process is reversible, we also have

$$dS = \frac{1}{T} dE .$$

Comparison yields

$$\frac{\partial \varphi}{\partial \rho} = \frac{1}{T} .$$

This is the law of black-body radiation. Thus, one can determine the law of black-body radiation from the function φ , and, vice versa, the function φ can be determined by integrating the former, considering that φ vanishes for $\rho = 0$.

§4. *Limiting law for the entropy of monochromatic radiation at low radiation density*

Though the existing observations of "black-body radiation" show that the law

$$\rho = a\nu^3 e^{-\beta \frac{\nu}{T}} \quad [23]$$

postulated by Mr. W. Wien for "black-body radiation" is not strictly valid, the law has been fully confirmed by experiment for large values of ν/T . We shall base our calculations on this formula, but will keep in mind that our results are valid within certain limits only. [24]

First of all, this formula yields

$$\frac{1}{T} = - \frac{1}{\beta \nu} \lg \frac{\rho}{a\nu^3} ,$$

and next, using the relation found in the preceding section,

$$\varphi(\rho, \nu) = - \frac{\rho}{\beta \nu} \left\{ \lg \frac{\rho}{a\nu^3} - 1 \right\} .$$

Now consider radiation of energy E whose frequency lies between ν and $\nu + d\nu$. Let the radiation occupy volume v . The entropy of this radiation is

$$S = v\varphi(\rho, \nu) d\nu = - \frac{E}{\beta \nu} \left\{ \lg \frac{E}{v a \nu^3 d\nu} - 1 \right\} . \quad [25]$$

If we restrict ourselves to investigating the dependence of the entropy on the volume occupied by the radiation and denote the entropy of radiation by S_0 when the latter occupies the volume v_0 , we obtain

$$S - S_0 = \frac{E}{\beta v} \lg \left[\frac{v}{v_0} \right] .$$

This equation shows that the entropy of a monochromatic radiation of sufficiently low density varies with the volume according to the same law as the entropy of an ideal gas or that of a dilute solution. The equation just found shall be interpreted in the following on the basis of the principle introduced into physics by Mr. Boltzmann, according to which the entropy of a system is a function of the probability of its state.

§5. *Molecular-theoretical investigation of the dependence of the entropy of gases and dilute solutions on the volume*

In calculating the entropy by molecular-theoretical methods, the word "probability" is often used in a sense that does not coincide with the definition of probability used in the probability calculus. In particular, the "cases of equal probability" are often stated hypothetically when the theoretical models applied are sufficiently definite to permit a deduction instead of a hypothetical statement. I will show in a separate paper that, when dealing with thermal processes, it is completely sufficient to use the so-called "statistical probability," and I hope that this will remove a logical difficulty that still hinders the implementation of Boltzmann's principle. Here, however, I shall only give its general formulation and its application to very special cases.

If it makes sense to talk about the probability of a state of a system, and if, further, each entropy increase can be conceived as a transition to a more probable state, then the entropy S_1 of a system is a function of the probability \mathcal{W}_1 of its instantaneous state. Therefore, if we have two systems S_1 and S_2 that do not interact with each other, we can put

$$\begin{aligned} S_1 &= \varphi_1(W_1) , \\ S_2 &= \varphi_2(W_2) . \end{aligned}$$

If these two systems are viewed as a single system of entropy S and probability W , we have

$$S = S_1 + S_2 = \varphi(W)$$

and

$$W = W_1 \cdot W_2 .$$

The last relation tells us that the states of the two systems are mutually independent events.

From these equations it follows that

$$\varphi(W_1 \cdot W_2) = \varphi_1(W_1) + \varphi_2(W_2)$$

and from this we get, finally,

$$\begin{aligned} \varphi_1(W_1) &= C \lg(W_1) + \text{const.} \\ \varphi_2(W_2) &= C \lg(W_2) + \text{const.} \\ \varphi(W) &= C \lg(W) + \text{const.} \end{aligned}$$

The quantity C is thus a universal constant; it follows from the kinetic theory of gases that its value is R/N , where the meaning of the constants R and N is the same as above. If S_0 denotes the entropy in some initial state of a system considered, and W the relative probability of a state having the entropy S , we obtain, in general,

[30]

$$S - S_0 = \frac{R}{N} \lg W .$$

First, we deal with the following special case. Let a volume v_0 contain a number (n) of movable points (e.g., molecules), which shall be the object of our consideration. The space may also contain any number of movable points of whatever kind. No assumptions shall be made about the law governing the motion of the points in the space except that, with regard to this motion,

no part of the space (and no direction) shall be distinguished from the others. The number of the (first-mentioned) movable points shall be so small that the effects of the points on each other can be disregarded.

This system, which might be, for example, an ideal gas or a diluted solution, possesses a certain entropy S_0 . Let us consider a part of the volume v_0 of magnitude v and let all n movable points be transferred into the volume v without any other change in the system. It is obvious that this state has a different value of entropy (S), and we now wish to determine the entropy difference with the aid of Boltzmann's principle.

We ask: How great is the probability of the last-mentioned state relative to the original one? Or: How great is the probability that at a randomly chosen instant of time all n independently movable points in a given volume v_0 will be contained (by chance) in volume v ?

Obviously, for this probability, which is a "statistical probability," one obtains the value

$$W = \left[\frac{v}{v_0} \right]^n ;$$

from this, by applying Boltzmann's principle, one obtains

$$S - S_0 = R \left[\frac{n}{N} \right] \lg \left[\frac{v}{v_0} \right] .$$

It is noteworthy that the derivation of this equation, from which the Boyle-Gay-Lussac law and the identical law of osmotic pressure can easily be obtained by thermodynamics,¹ does not require any assumptions about the law governing the motion of the molecules.

¹If E is the energy of the system, we obtain

[31]

hence

$$- d(E - TS) = pdv = TdS = R \frac{n}{N} \frac{dv}{v} ;$$

$$pv = R \frac{n}{N} T .$$

§6. *Interpretation of the expression for the dependence of the entropy of monochromatic radiation on volume according to Boltzmann's principle*

In §4 we found the following expression for the dependence of the entropy of monochromatic radiation on volume:

$$S - S_0 = \frac{E}{\beta\nu} \lg \left[\frac{v}{v_0} \right] .$$

If we write this formula in the form

$$S - S_0 = \frac{R}{N} \lg \left[\left[\frac{v}{v_0} \right]^{\frac{N}{R}} \frac{E}{\beta\nu} \right]$$

and compare it with the general formula expressing the Boltzmann principle

$$S - S_0 = \frac{R}{N} \lg W ,$$

we arrive at the following conclusion:

If monochromatic radiation of frequency ν and energy E is enclosed (by reflecting walls) in the volume v_0 , the probability that at a randomly chosen instant the entire radiation energy will be contained in the portion v of the volume v_0 is

$$W = \left[\frac{v}{v_0} \right]^{\frac{N}{R} \frac{E}{\beta\nu}}$$

From this we further conclude:

Monochromatic radiation of low density (within the range of validity of Wien's radiation formula) behaves thermodynamically as if it consisted of mutually independent energy quanta of magnitude $R\beta\nu/N$.

We also wish to compare the mean value of the energy quanta of "black-body radiation" with the mean kinetic energy of the center-of-mass motion of a molecule at the same temperature. The latter is $\frac{3}{2}(R/N)T$, while the mean value of the energy quantum obtained on the basis of the Wien formula is

[32]

$$\frac{\int_0^\infty \alpha \nu^3 e^{-\frac{\beta \nu}{T}} d\nu}{\int_0^\infty \frac{N}{R\beta \nu} \alpha \nu^3 e^{-\frac{\beta \nu}{T}} d\nu} = 3 \frac{R}{N} T .$$

If, with regard to the dependence of its entropy on volume, a monochromatic radiation (of sufficiently low density) behaves like a discontinuous medium consisting of energy quanta of magnitude $R\beta\nu/N$, then it seems reasonable to investigate whether the laws of generation and conversion of light are also so constituted as if light consisted of such energy quanta. We will now consider this question.

[33]

§7. On Stokes' rule

Let monochromatic light be converted by photoluminescence to light of another frequency, and let us assume in accordance with the result just obtained that both the producing and the produced light consist of energy quanta of magnitude $(R/N)\beta\nu$, where ν denotes the pertinent frequency. The conversion process is then to be interpreted as follows. Each producing energy quantum of frequency ν_1 is absorbed and—at least at a sufficiently low distribution density of the producing energy quanta—by itself gives rise to the generation of a light quantum of frequency ν_2 ; possibly the absorption of the producing light quantum might also be accompanied by the simultaneous generation of light quanta of frequencies ν_3, ν_4 , etc., as well as of energy of some other kind (e.g., heat). It makes no difference by what kind of intermediary processes this end result is mediated. If the photoluminescent substance is not to be regarded as a permanent source of energy, then, according to the energy principle, the energy of a produced energy quantum cannot be greater than that of a producing light quantum; hence we must have

$$\frac{R}{N} \beta \nu_2 \leq \frac{R}{N} \beta \nu_1 ,$$

or

$$\nu_2 \leq \nu_1 .$$

This is the well-known Stokes' rule.

It should be especially emphasized that, according to our conception, at weak illumination the produced amount of light must be proportional to the intensity of the exciting light, because each exciting energy quantum will induce an elementary process of the kind indicated above, independent of the action of the other exciting energy quanta. In particular, no lower limit will exist for the intensity of the exciting light below which the light would [34] be unable to act as an exciter of light.

According to the conception of the phenomena expounded, deviations from Stokes' rule are conceivable in the following cases: [35]

1. When the number of simultaneously converting energy quanta per unit volume is so large that an energy quantum of the light produced could obtain its energy from several producing quanta;
2. When the producing (or produced) light does not have the same energy properties that obtain for "black-body radiation" within the range of validity of Wien's law as, for example, when the exciting light is produced by a body of such high temperature that Wien's law is no longer valid for the pertinent wavelength.

The latter possibility deserves special attention, for according to the conception expounded above it is not impossible that even in great dilutions the energetic behavior of a "non-Wien radiation" differs from that of a "black-body radiation" that is within the range of validity of Wien's law.

§8. *On the generation of cathode rays by illumination of solid bodies*

The usual conception, that the energy of light is continuously distributed over the space through which it travels, meets with especially great difficulties when one attempts to explain the photoelectric phenomena; these difficulties are presented in a pioneering work by Mr. Lenard.¹ [36]

According to the conception that the exciting light consists of energy quanta of energy $(R/N)\beta\nu$, the production of cathode rays by light can be conceived in the following way. The body's surface layer is penetrated by

¹P. Lenard, *Ann. d. Phys.* 8 (1902): 169 and 170. [37]

energy quanta whose energy is converted at least partially to kinetic energy of electrons. The simplest possibility is that a light quantum transfers its entire energy to a single electron; we will assume that this can occur.

However, we will not exclude the possibility that the electrons absorb only a part of the energy of the light quanta. An electron provided with kinetic energy in the interior of the body will have lost a part of its kinetic energy [38] by the time it reaches the surface. In addition, it will have to be assumed that in leaving the body, each electron has to do some work P (characteristic for the body). The greatest perpendicular velocity on leaving the body will be that of electrons located directly on the surface and excited perpendicular to it. The kinetic energy of such electrons is

$$\frac{R}{N} \beta \nu - P .$$

If the body is charged to the positive potential Π and is surrounded by conductors of zero potential, and if Π is just sufficient to prevent a loss of electricity of the body, we must have

$$\Pi \epsilon = \frac{R}{N} \beta \nu - P ,$$

[39] where ϵ denotes the electric mass of the electron, or

$$\Pi E = R \beta \nu - P' ,$$

where E denotes the charge of one gram-equivalent of a univalent ion and P' is the potential of this quantity of negative electricity with respect to the body.¹

[41] If one sets $E = 9.6 \times 10^3$, then $\Pi \cdot 10^{-8}$ is the potential in volts that the body acquires during irradiation in the vacuum.

To see whether the relation derived agrees with experience in order of magnitude, we put $P' = 0$, $\nu = 1.03 \times 10^{15}$ (which corresponds to the limit

[40] ¹If one assumes that the release of the individual electron from a neutral molecule by light must be accompanied by the expenditure of some work, one does not have to change anything in the above relation; but then P' is to be considered as the sum of two summands.

of the solar spectrum toward the ultraviolet) and $\beta = 4.866 \times 10^{-11}$. We [42]
 obtain $\Pi \cdot 10^7 = 4.3$ volt, a result that agrees in order of magnitude with the
 results of Mr. Lenard.¹

If the formula derived is correct, then Π , presented as a function of
 the frequency of the exciting light in Cartesian coordinates, must be a
 straight line whose slope is independent of the nature of the substance [44]
 investigated.

As far as I can see, our conception does not conflict with the proper-
 ties of the photoelectric effect observed by Mr. Lenard. If each energy
 quantum of the exciting light transmits its energy to electrons independent of
 all others, then the velocity distribution of the electrons, i.e., the quality
 of the cathode rays produced, will be independent of the intensity of the
 exciting light; on the other hand, under otherwise identical circumstances,
 the number of electrons leaving the body will be proportional to the intensity
 of the exciting light.²

Remarks similar to those regarding the expected deviations from Stokes'
 rule apply to the expected limits of validity of the laws mentioned above.

In the foregoing it has been assumed that the energy of at least some of
 the energy quanta of the producing light is transmitted completely to one
 single electron each. If this obvious assumption is not made, instead of the
 above equation one obtains the following one:

$$\Pi E + P' \leq R\beta\nu .$$

For the cathode luminescence, which constitutes the inverse process of
 that discussed above, one obtains by a consideration analogous to that above [45]

$$\Pi E + P' \geq R\beta\nu .$$

For the substances investigated by Mr. Lenard, PE is always considerably [46]
 larger than $R\beta\nu$ because the potential difference the cathode rays must

¹P. Lenard, *Ann. d. Phys.* 8 (1902): 165 and 184, Table I, Fig. 2. [43]

²P. Lenard, *loc. cit.*, p. 150 and pp. 166-168.

traverse in order to produce light that is just visible amounts to several hundred volts in some cases, and to thousands of volts in others.¹ We must therefore assume that the kinetic energy of one electron is used for the production of many quanta of light energy.

[48] §9. *On the ionization of gases by ultraviolet light*

We will have to assume that in the ionization of a gas by ultraviolet light one quantum of light energy is used for the ionization of one molecule of gas. From this it follows that the work of ionization (i.e., the work theoretically required for ionization) of one molecule cannot be greater than the energy of one effective quantum of light absorbed. If J denotes the (theoretical) ionization work per gram-equivalent, we must have

$$R\beta\nu \geq J .$$

[49] However, according to measurements by Lenard, the largest effective wavelength for air is about 1.9×10^{-5} cm, hence

$$R\beta\nu = 6.4 \times 10^{12} \text{ erg} \geq J .$$

An upper limit for the work of ionization can also be obtained from the ionization potentials in rarefied gases. According to J. Stark² the smallest measured ionization potential (at platinum anodes) for air is about 10 volts.³ Thus one obtains 9.6×10^{12} as the upper limit for J , which is almost equal to the value we have just found. There is still another consequence, whose verification by experiment seems to me of great importance. If each absorbed quantum of light energy ionizes one molecule, then the following relation

[47] ¹P. Lenard, *Ann. d. Phys.* 12 (1903): 469.

[50] ²J. Stark, *Die Elektrizität in Gasen*, p. 57. Leipzig, 1902

³In the interior of the gases the ionization potential of negative ions is five times larger, however.

must hold between the quantity of light absorbed L and the number j of gram-molecules ionized by it:

$$j = \frac{L}{h\nu} . \quad [51]$$

If our conception corresponds to reality, this relation must apply to all gases that (at the relevant frequency) display no noticeable absorption that is not accompanied by ionization.

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